Questions

Q1.

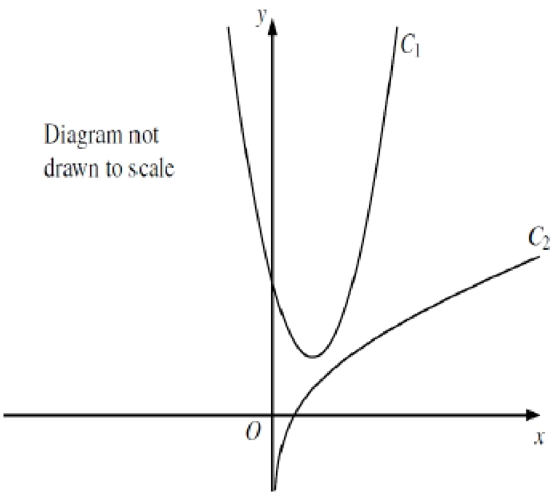


Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

(Total for question = 8 marks)

Q2.

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(Total for question = 5 marks)

Q3.

A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P

and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

(Total for question = 10 marks)

Q4.

The curve *C* has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where *k* is a constant.

(a) Sketch *C* stating the equation of the horizontal asymptote.

(3)

The line *l* has equation $y = -2x + 5$

(b) Show that the *x* coordinate of any point of intersection of *l* with *C* is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of *k* for which *l* is a tangent to *C*.

(3)

(Total for question = 8 marks)

Q5.

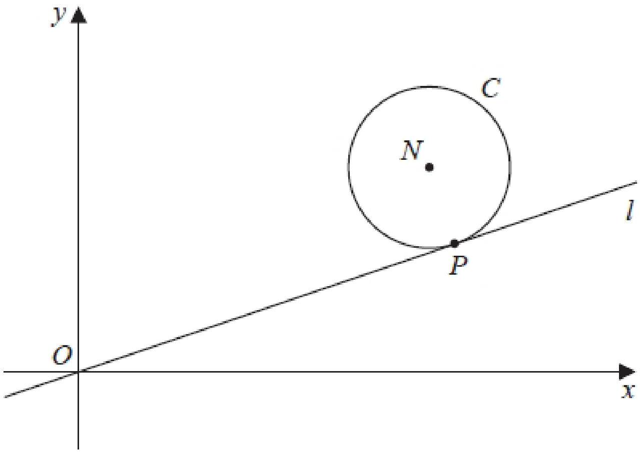


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

(Total for question = 9 marks)

Q6.

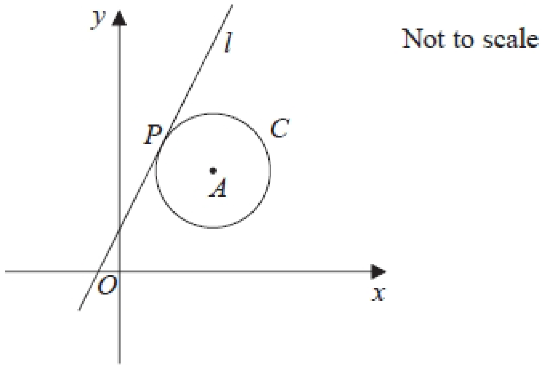


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is -2	M1	1.1b
	Normal gradient is $-1/m = 1/2$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

Notes

M1: Differentiates correctly
M1: Substitutes $x = ½$ to find gradient (may make a slip)
M1: Uses negative reciprocal gradient
A1: Correct equation for normal
M1:Attempts to eliminate y to find an equation in x
M1: Attempts to solve their equation using exp
M1: Uses their x value to find y
A1: Any correct exact form.

Q2.

Question	Scheme	Marks	AOs
	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.
Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$
It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Q3.

Question	Scheme		Marks	AOs
(a)	Way 1: Finds circle equation $(x\pm2)^2+(y\mp6)^2=(10\pm(-2))^2+(11\mp6)^2$	Way 2: Finds distance between $(-2,6)$ and (10, 11)	M1	3.1a
	Checks whether (10,1) satisfies their circle equation	Finds distance between $(-2,6)$ and (10, 1)	M1	1.1b
	Obtains $(x+2)^2+(y-6)^2=13^2$ and checks that $(10+2)^2+(1-6)^2=13^2$ so states that (10,1) lies on C *	Concludes that as distance is the same (10, 1) lies on the circle C *	A1*	2.1
			(3)	
(b)	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or $-$23		A1	1.1b
	Way 1: Deduces gradient of second tangent	Way 2: Deduces midpoint of PQ from symmetry ((0,6))	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ=35+23=58*		A1*	1.1b
			(7)	
				(10 marks)

Notes

(a) Way 1 and Way 2:

M1 : Starts to use information in question to find equation of circle or radius of circle

M1 : Completes method for checking that (10, 1) lies on circle

A1*: Completely correct explanation with no errors concluding with statement that circle passes through (10, 1)

(b) M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ or $\frac{12}{5}$) This is referred to as m' in the next note.

M1: Attempts $y-11=their\left(-\frac{12}{5}\right)(x-10)$ or $y-1=their\left(\frac{12}{5}\right)(x-10)$ and puts $x=0$, or uses vectors to find intercept e.g. $\frac{y-11}{10}=-m'$

A1: One correct intercept 35 or $-$23

Qu 17(b) continued

Way 1:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept e.g. $\frac{11-y}{10} = m'$

Way 2:

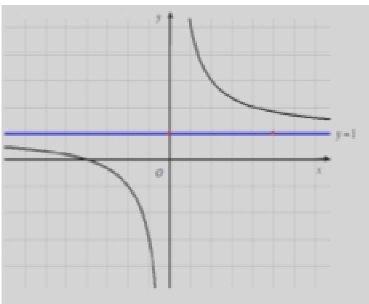
M1: Finds midpoint of PQ from symmetry. (This is at (0,6))

M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. $35 - 6 = 29$ then $6 - 29 = -23$ so second intercept is at $(-23, 0)$

Ways 1 and 2:

A1*: Obtain 58 correctly from a valid method.

Q4.

Question	Scheme	Marks	AOs
(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y = 1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x} + 1 = -2x + 5$	M1	1.1b
	$(\times x) \Rightarrow k^2 + 1x = -2x^2 + 5x \Rightarrow 2x^2 - 4x + k^2 = 0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2 - 4ac = 0$	M1	3.1a
	$8k^2 = 16$	A1	1.1b
	$k = \pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes
(a) **M1:** For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification) **A1:** Correct shape and position for both branches. It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour **B1:** Asymptote given as $y=1$. This could appear on the diagram or within the text. Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate. **(b)** **M1:** Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x **A1*:** Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips. Condone if the order of the terms are different $2x^2+k^2-4x=0$ **(c)** **M1:** Deduces that $b^2-4ac=0$ or equivalent for **the given equation.** If a, b and c are stated only accept $a=2, b=\pm 4, c=k^2$ so $4^2-4\times 2\times k^2=0$ Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow "1-\frac{1}{2}k^2"=0$ **A1:** $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct **A1:** $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows **M1:** Sets the gradient of the curve $=-2 \Rightarrow -\frac{k^2}{x^2}=-2 \Rightarrow x=(\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$ **A1:** $2k^2=(\pm)2\sqrt{2}k$ oe **A1:** $k=\pm\sqrt{2}$

Q5.

Question	Scheme	Marks	AOs
(a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7, 4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point P. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2}, \frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2}, \frac{5}{2}\right)$ and $(7, 4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets C using a vector approach

M1: For a full method leading to k . Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both b and c are dependent upon k.** The terms in x^2 and x must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in k.
Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding k

A1: $k=\frac{10}{3}$ only

Q6.

Question	Scheme	Marks	AOs
(a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $$y-5=-\frac{1}{2}(x-7)$$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)

(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=(\sqrt{20})^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or, **in the alternative**, for solving their $(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding k

A1: $k=-19$ only

Other method are possible using trigonometry.